Outline

- My slides: http://adlainewson.com

1. Consumer preferences and utility functions
2. Indifference Curves
3. Marginal Rate of Substitution (MRS)
4. Budget constraints
5. Consumer choice (time permitting)

Consumer Preferences

Notation: read $a \succsim b$ as "a is at least as good as b."
The model of consumer behaviour requires the following assumptions:

1. Completeness: $a \succsim b$ or $b \succsim a \ \forall a, b \in X$
2. Transitivity: $a \succsim b$, $b \succsim c \implies a \succsim c$
3. Local nonsatiation: can always think of a better alternative (eg: two ferraris instead of one)

Utility functions

With these assumptions, we can construct a function $u(\cdot)$ such that

$$a \succsim b \textit{ if and only if } u(a) \geq u(b)$$

This lets us work with numbers instead of ‘things’ (ferraris, ice cream, etc). Since utility functions are just representations of $\succsim$, they are *ordinal*.

Indifference Curves

The *level curve* of a function is the collection of points $\{x, y\}$ such that

$$f(x, y) = c$$

where c is some number.

Example

For example, if $f(x, y)$ describes the temperature $d = \sqrt{x^2 + y^2}$ feet away from a fire, then the level curve $f(x, y) = 10$ is the circle around the fire where temperature is 10 degrees.

Similarly, $u(x, y) = v$ is the collection of x and y such that utility is constant and equal to v.

IC of Utility Functions

Let's look at the IC's of

$$u(x,y) = 2x + y$$

and

$$u(x,y) = 4\sqrt{xy}$$

and

$$u(x,y) = \min\{x,y\}$$

Probably all of the utility functions you see will behave similarly to the above three examples.

Marginal Rate of Substitution

Define the MRS as

$$
\begin{aligned}
MRS &= -\frac{\partial u(x,y)/\partial x}{\partial u(x,y)/\partial y} \\
&\equiv -\frac{u_x}{u_y}
\end{aligned}
$$

Let's find the MRS for the examples on the last slide.

MRS: Derivation

Where does the MRS come from? Consider the IC at c

$$\begin{aligned} c &= u(x,y) \\ dc &= \frac{\partial u(x,y)}{\partial x}dx + \frac{\partial u(x,y)}{\partial y}dy \end{aligned}$$

The second line is the total derivative of the first line. Along the IC, by definition, $dc = 0$. Therefore,

$$\begin{aligned} 0 &= u_x dx_{IC} + u_y dy_{IC} \\ \frac{dy_{IC}}{dx_{IC}} &= -\frac{u_x}{u_y} \end{aligned}$$

So the MRS is the slope of the indifference curve!

Characterizing MRS

- Usually MRS is decreasing in magnitude along the IC - why?
  - Two possible reasons.
  - Math: Differentiate MRS w.r.t x holding y fixed...
  - Intuition for reason 1: Suppose x is pepsi and y is hot dogs...
  - Intuition for reason 2: Suppose x is ice creamand y is cashmoney...
- Perfect substitute, perfect compliment examples.

Budget Constraints

Here's the 'scarcity' part from the textbook definition of economics... we all have budget constraints.

Three components of a budget constraint: goods, income, prices.

Let's look at some examples...

Consumer Choice

As always, the idea in completing the consumer choice problem will be using a condition like $MR = MC$ from 101. What are MR and MC in this model? MU_x and p_x and MU_y and p_y. So our first guess might be something like using

$$\frac{\partial u(x^*, y^*)}{\partial x} = p_x$$
$$\frac{\partial u(x^*, y^*)}{\partial y} = p_y$$

but the problem here is that the LHS is measured in utility and the RHS is measured in dollars; there's a missing 'price' of utility in terms of dollars on the RHS.

Consumer Choice

Call this price λ. Then we have two equations:

$$\frac{\partial u(x^*, y^*)}{\partial x} = \lambda p_x$$
$$\frac{\partial u(x^*, y^*)}{\partial y} = \lambda p_y$$

But we don't know λ, so let's divide those two expressions to get:

$$\frac{u_x}{u_y} = \frac{p_x}{p_y}$$

Look familiar? We can then use the budget constraint to have 2 equations (the one above and the budget constraint) and 2 unknowns (x^*, y^*).

The one key point here is that this only works for **interior solutions** $(x^* > 0,\ y^* > 0)$.

Consumer Choice

Example: Based on Ch3, Q15. Find the solutions x^*, y^* to the Consumer Problem (CP):

$$u(x,y) = \alpha xy$$

s.t. $4 \geq 4y + x$.
Note that here I'm not restricting x, y to be integers.
Solution: Since the solution will be interior (why?), we have

$$MRS = -\frac{\alpha y}{\alpha x} = -\frac{y}{x}$$
$$-\frac{p_x}{p_y} = -\frac{1}{4}$$

so $4y = x$. Using the Budget Constraint (BC) at equality: $4 = 4(\frac{1}{4}x) + x$, which gives $x = 2$, $y = \frac{1}{2}$. Why doesn't the solution depend on α?
EDIT: first version used $\frac{p_y}{p_x}$ rather than $\frac{p_x}{p_y}$.