

1. Find a Nash equilibrium to the following game:

		Player 2		
		L	C	R
Player 1	U	(10, 4)	(2, 10)	(4, 5)
	M	(5, 6)	(7, 2)	(3, 7)
	D	(4, 9)	(6, 4)	(9, 3)

No pure strategy Nash Eq.

No strictly dominated strategies.

Look for a mixed strategy Nash equilibrium where both players play each of the three available actions with positive probability.

Let player 1 choose actions with probabilities:

action	probability
U	$\alpha$
M	$\lambda$
D	$(1 - \alpha - \lambda)$

Let player 2 choose actions with probabilities:

action	probability
L	$\beta$
C	$\delta$
R	$(1 - \beta - \delta)$

For player 1 :

$$\begin{aligned} \mathbb{E}_1 \{U\} &= \mathbb{E}_1 \{M\} \\ 10\beta + 2\delta + 4(1 - \beta - \delta) &= 5\beta + 7\delta + 3(1 - \beta - \delta) \\ 5\beta + 1 - \beta - \delta &= 5\delta \\ \beta &= \frac{3}{2}\delta - \frac{1}{4} \end{aligned}$$

$$\begin{aligned} \mathbb{E}_1 \{U\} &= \mathbb{E}_1 \{D\} \\ 10\beta + 2\delta + 4(1 - \beta - \delta) &= 4\beta + 6\delta + 9(1 - \beta - \delta) \\ 11\beta &= 5 - \delta \end{aligned}$$

substitute the expression for  $\beta$

$$\begin{aligned} 11 \left( \frac{3}{2}\delta - \frac{1}{4} \right) &= 5 - \delta \\ \delta &= \frac{31}{70} \\ \beta &= \frac{3}{2} \left( \frac{31}{70} \right) - \frac{1}{4} = \frac{29}{70} \\ (1 - \beta - \delta) &= \frac{10}{70} \end{aligned}$$

For player 2 :

$$\begin{aligned}\mathbb{E}_2 \{L\} &= \mathbb{E}_2 \{C\} \\ 4\alpha + 6\lambda + 9(1 - \alpha - \lambda) &= 10\alpha + 2\lambda + 4(1 - \alpha - \lambda) \\ 4\lambda + 5(1 - \alpha - \lambda) &= 6\alpha \\ \lambda &= 5 - 11\alpha\end{aligned}$$

$$\begin{aligned}\mathbb{E}_2 \{L\} &= \mathbb{E}_2 \{R\} \\ 4\alpha + 6\lambda + 9(1 - \alpha - \lambda) &= 5\alpha + 7\lambda + 3(1 - \alpha - \lambda) \\ 6 &= 7\alpha + 7\lambda \\ &\text{substitute expression for } \lambda \\ \alpha &= \frac{29}{70} \\ \lambda &= \frac{31}{70} \\ (1 - \alpha - \lambda) &= \frac{10}{70}\end{aligned}$$

$$\left\{ \left( \begin{array}{l} U \text{ with prob } \frac{29}{70} \\ M \text{ with prob } \frac{31}{70} \\ D \text{ with prob } \frac{10}{70} \end{array} \right), \left( \begin{array}{l} L \text{ with prob } \frac{29}{70} \\ C \text{ with prob } \frac{31}{70} \\ R \text{ with prob } \frac{10}{70} \end{array} \right) \right\}$$