PS1 Q4 SOLUTION, ADLAI NEWSON

A particular street corner is a popular location for food carts to sell lunch to people working in the area. Two vendors must each decide in the morning whether to operate carts on a particular day or stay at home. One vendor operates a small size Hot Dog cart (H), the other vendor operates a Thai food cart (T). The Thai food vendor can choose to bring either a large cart or a small cart. There is a higher cost of operating the large cart (additional person), but the Thai food vendor will serve a larger share of the customers. There is fixed demand for 40 lunches at \$10 each (Total market sales/revenue will be \$400). The cost of operating a small cart is \$100 (for both H and T), while the cost of operating the large Thai food cart is \$150: When only one vendor shows up to the street corner they will get 100% of the sales. If both show up with small carts the two vendors split sales evenly. When the Thai food vendor shows up with a large cart they will get 75% of the sales (25% to the Hot Dog vendor). Note: decisions are made simultaneously.

part a. Can this economic interaction be modeled as a game? If so, identify all the elements that make this a well-defined game.

Answer (1pt): Yes. Formal game definition for this game:

- Players: H, T
- Rules:
  - Moves are simultaneous.
  - Actions for H are Small cart S_H, or stay home, H_H
  - Actions for T are Large cart L_T, Small cart S_T, or stay home, H_T
- Outcomes: $\{S_H L_T, S_H S_T, S_H H_T, H_H L_T, H_H S_T, H_H H_T\}$
- Payoffs:
  - $\pi_H(H_H, L_T) = \pi_H(H_H, S_T) = \pi_H(H_H, H_T) = \pi_T(H_H, H_T) = \pi_T(S_H, H_T) = 0$
  - $\pi_H(S_H, H_T) = \pi_T(H_H, S_T) = 300$
  - $\pi_H(S_H, S_T) = \pi_T(S_H, S_T) = 100$
  - $\pi_T(H_H, L_T) = 250$
  - $\pi_T(S_H, L_T) = 150$
  - $\pi_H(S_H, L_T) = 0$

part b. Show the game matrix for this interaction.

Answer (2pts):

T\H	S_H	H_H
L_T	$150, 0$	$250, 0$
S_T	$100, 100$	$300, 0$
H_T	$0, 300$	$0, 0$

Obviously any permutation of rows and columns or matrix transpose is also acceptable.

part c. Does either player have any strictly dominant strategies?

Answer (.5pt). No.

part d. Does either player have any strictly dominated strategies?

Answer (.5pt). Yes. H_T is strictly dominated by L_T and S_T.

part e. Identify all Nash equilibria to this game.

Answer (1pt). We look for all possible mixed strategy equilibria (which will include the pure strategy equilibrium). Let $\alpha = \Pr(H \text{ plays } S_H)$, $\beta = \Pr(T \text{ plays } L_T)$. Then

$$
\begin{aligned}
\mathrm{E}\pi_H(S_H) &= \mathrm{E}\pi_H(H_H) \\
100\,(1-\beta) &= 0 \\
&\implies \beta = 1
\end{aligned}
$$

For $\beta = 1$ to be supported by a rational player T, we require

$$
\begin{aligned}
\mathrm{E}\pi_T(L_T) &\geq \mathrm{E}\pi_T(S_T) \\
150\alpha + 250\,(1-\alpha) &\geq 100\alpha + 300\,(1-\alpha) \\
50\alpha &\geq 50\,(1-\alpha) \\
&\implies \alpha \geq \frac{1}{2}
\end{aligned}
$$

Therefore the set of Nash equilibria is

$$
\{L_T, (S_H \text{ wp } \alpha,\ H_H \text{ wp } 1-\alpha)\} \qquad \forall \alpha \in \left[\frac{1}{2}, 1\right]
$$