Econ 221 (002 & 004)
Winter Session Term I, 2015
M. Vaney

Problem Set 4

This assignment is due Friday November 27 by 1:30pm (my office - BUTO 1011). Remember to write your NAME, STUDENT NUMBER, DISCUSSION GROUP and the name of your T.A. at the top of the front page of the problem set and to staple your problem set together before handing it in. Failure to do so may result in loss of marks on the assignment.
All work must be shown. Answer ALL questions. At least one question will be graded.

1. A small town has 100 cabs. 70 cabs are blue (Blue Cab Company) and 30 cabs are green (Green Cab Company). There is a hit-and-run accident at night (pedestrian injury, not fatality) and an eyewitness present at the scene of the accident claims it was a green cab that struck the pedestrian. No criminal charges are laid, but the injured party does bring a civil lawsuit against the Green Cab Company to recover costs associated with hospitalization and lost wages. The witness is called to testify ('I saw a green cab strike the pedestrian'). The witness is also subjected to a vision test and it is found that under conditions similar to those on the night of the accident, the witness can correctly identify a cab from other types of vehicles and the witness can correctly distinguish the colour of the cab (blue or green) $X\%$ of the time. The judge in the case will find in favour of the plaintiff if the beliefs of the judge assign a probability of greater than 50% to a green cab being involved in the hit-and-run.

   (a) If $X\% = 100\%$ (the witness always identifies the correct colour of the cab), how should the judge rule in this case?

   (b) If $X\% = 75\%$ (the witness correctly identifies the colour of the cab 75% of the time), how should the judge rule in this case?

   (c) Suppose that evidence is presented that show that on the night in question there were only 60 of the 70 blue cabs in service and 25 of the 30 green cabs in service. Would this lead to a change in the ruling of the judge ($X\% = 75\%$).

2. You are a contestant in a game show. Three doors are presented (unopened). Behind each door is a prize. Only one of the prizes is of any value, $V > 0$. Both of the other prizes are essentially useless and of no value. The arrangement of prizes behind doors is essentially random. You must select one of the 3 doors. Before your door is opened the host will show you what is behind one of the two doors that have not been selected. Two rules that are followed by the host: *never* open the door selected by the contestant and *always* open a door with a useless ($V = 0$) prize, *never* open the door with the good ($V > 0$) prize. The host will now offer you the option of sticking to your original choice of door *or* switching to a different door. You would never switch your choice to the door that was opened because you know it contains a useless ($V = 0$) prize. Let D_i ($i = 1, 2, 3$) be the location of the good ($V > 0$) prize. Let R_i ($i = 1, 2, 3$) the door that is opened/revealed. You start by choosing door 2. After your selection of door 2, the host opens door 1 to reveal a useless ($V = 0$) prize.

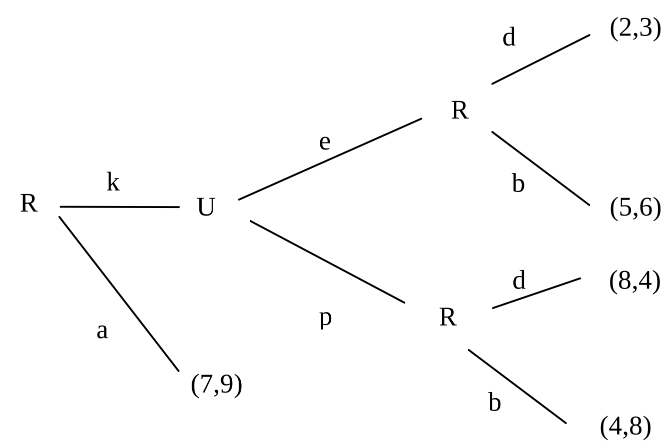
(a) What is the unconditional probability of the good ($V > 0$) prize being behind the chosen door: $\mathbb{P}(D_2)$? A door that was not chosen? $\mathbb{P}(D_1)$? $\mathbb{P}(D_3)$? (Before learning that door 1 will be opened to reveal a useless ($V = 0$) prize).

(b) What is the conditional/updated probability of the good ($V > 0$) prize being behind door 1? behind door 2? behind door 3?

(c) What is the optimal strategy of the contestant with respect to sticking to the original choice of door or switching to a different door? How does the probability of winning the good ($V > 0$) prize change for the contestant?

(d) Suppose that the same game is played with 4 doors, 1 good ($V > 0$) prize and 3 useless ($V = 0$) prizes. Work through (a), (b), and (c) for this case.

3. The U.N. (player U) is concerned with the weapons program of a Rogue dictator (player R). The U.N. has sent weapons inspectors to the country, if R allows (a) the inspectors to carry out their program then the game will come to an end. If R chooses to kick out (k) the inspectors, then U must choose how to respond (Escalate (e) or take a Passive role (p)) and R will then choose whether to remain defiant (d) or Back down (b). The game tree can be shown as follows (payoff to R, payoff to U):



(a) Find the subgame perfect Nash equilibrium to this game.

(b) Suppose that the Rogue dictator may be one of two types. The Sane type, R_S has payoffs as described in the game tree above. There is also a Crazy type, R_C. The Crazy type likes confrontation and likes to be defiant. For the Crazy type, the game is as above except for two changes to the payoffs of R_C:

1. if R_C allows the inspectors then R_C will receive a payoff of 3 (not 7)

2. when R plays k, U plays e and R plays d the payoff to R_C is 9 instead of 2.

The Rogue dictator knows his type. The U.N. is unable to observe the type but does know that the probability of the dictator being Sane is $\pi = \frac{1}{2}$. Use Nature as an additional player to draw the game tree.

(c) Consider an equilibrium where R_C plays a single action, but both R_S and U follow mixed strategies. R_S plays k with probability α (and a with probability $1 - \alpha$)

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Problem Set 4 - Solutions

1. Bayes

(a) $\mathbb{P}(G|W_G) = 1$ and $\mathbb{P}(B|W_B)$

$$\begin{aligned}\mathbb{P}(G|W_G) &= \frac{\mathbb{P}(W_G|G)\,\mathbb{P}(G)}{\mathbb{P}(W_G|G)\,\mathbb{P}(G) + \mathbb{P}(W_G|B)\,\mathbb{P}(B)} \\ &= \frac{(1)(\frac{3}{10})}{(1)(\frac{3}{10}) + (0)(\frac{7}{10})} = 1\end{aligned}$$

(b) $\mathbb{P}(G|W_G) = \frac{3}{4}$, $\mathbb{P}(G|W_B) = \frac{1}{4}$

$$\begin{aligned}\mathbb{P}(G|W_G) &= \frac{\mathbb{P}(W_G|G)\,\mathbb{P}(G)}{\mathbb{P}(W_G|G)\,\mathbb{P}(G) + \mathbb{P}(W_G|B)\,\mathbb{P}(B)} \\ &= \frac{(\frac{3}{4})(\frac{3}{10})}{(\frac{3}{4})(\frac{3}{10}) + (\frac{1}{4})(\frac{7}{10})} = \frac{9}{16} > 50\%\end{aligned}$$

(c) $\mathbb{P}(G|W_G) = \frac{3}{4}$, $\mathbb{P}(G|W_B) = \frac{1}{4}$ and now $\mathbb{P}(G) = \frac{5}{17}, \mathbb{P}(B) = \frac{12}{17}$

$$\begin{aligned}\mathbb{P}(G|W_G) &= \frac{\mathbb{P}(W_G|G)\,\mathbb{P}(G)}{\mathbb{P}(W_G|G)\,\mathbb{P}(G) + \mathbb{P}(W_G|B)\,\mathbb{P}(B)} \\ &= \frac{(\frac{3}{4})(\frac{5}{17})}{(\frac{3}{4})(\frac{5}{17}) + (\frac{1}{4})(\frac{12}{17})} = \frac{5}{9} > 50\%\end{aligned}$$

2. Monte Hall

(a) $\mathbb{P}(D_i) = \frac{1}{3}$

(b) Contestant chooses Door 2, Door 1 is opened: R_1

$\mathbb{P}(D_1|R_1) = 0$. We know this because the door that is opened is never the door with the good $(V > 0)$ prize.

For $\mathbb{P}(D_2|R_1)$. Use Baye's

$$\mathbb{P}(D_2|R_1) = \frac{\mathbb{P}(R_1|D_2)\,\mathbb{P}(D_2)}{\mathbb{P}(R_1|D_1)\,\mathbb{P}(D_1) + \mathbb{P}(R_1|D_2)\,\mathbb{P}(D_2) + \mathbb{P}(R_1|D_3)\,\mathbb{P}(D_3)}$$

$\mathbb{P}(R_1|D_2) = \frac{1}{2}$: If the good prize is behind door 2 (which is the chosen door) then either door 1 or door 3 can be opened.

$\mathbb{P}(R_1|D_1) = 0$: The host never reveals the location of the good prize.

$\mathbb{P}(R_1|D_3) = 1$: If the good prize is behind door 3 and contestant chooses door 2 then door 1 is the only door the host can open.

$$\mathbb{P}(D_2|R_1) = \frac{(\frac{1}{2})(\frac{1}{3})}{(0)(\frac{1}{3}) + (\frac{1}{2})(\frac{1}{3}) + (1)(\frac{1}{3})} = \frac{1}{3}$$

And this means that $\mathbb{P}(D_3|R_1) = 1 - \mathbb{P}(D_1|R_1) - \mathbb{P}(D_2|R_1) = 1 - 0 - \frac{1}{3} = \frac{2}{3}$

(c) Contestant should switch from door 2 to door 3.

(d) $\mathbb{P}(D_i) = \frac{1}{4}$

Contestant chooses Door 2, Door 1 is opened: R_1

$\mathbb{P}(D_1|R_1) = 0$. We know this because the door that is opened is never the door with the good ($V > 0$) prize.

For $\mathbb{P}(D_2|R_1)$. Use Baye's

$$\mathbb{P}(D_2|R_1) = \frac{\mathbb{P}(R_1|D_2)\mathbb{P}(D_2)}{\mathbb{P}(R_1|D_1)\mathbb{P}(D_1) + \mathbb{P}(R_1|D_2)\mathbb{P}(D_2) + \mathbb{P}(R_1|D_3)\mathbb{P}(D_3) + \mathbb{P}(R_1|D_4)\mathbb{P}(D_4)}$$

$\mathbb{P}(R_1|D_2) = \frac{1}{3}$: If the good prize is behind door 2 (which is the chosen door) then either door 1, door 3, or door 4 can be opened.

$\mathbb{P}(R_1|D_1) = 0$: The host never reveals the location of the good prize.

$\mathbb{P}(R_1|D_3) = \frac{1}{2}$: If the good prize is behind door 3 and contestant chooses door 2 then either door 1 or door 4 can be opened.

$\mathbb{P}(R_1|D_4) = \frac{1}{2}$: If the good prize is behind door 4 and contestant chooses door 2 then either door 1 or door 3 can be opened.

$$\mathbb{P}(D_2|R_1) = \frac{(\frac{1}{3})(\frac{1}{4})}{(0)(\frac{1}{4}) + (\frac{1}{3})(\frac{1}{4}) + (\frac{1}{2})(\frac{1}{4}) + (\frac{1}{2})(\frac{1}{4})} = \frac{1}{4}$$

The probability that the contestant will win the prize if they stick to the original choice will be $\frac{1}{4}$.

Compare this with switching to door 3:

$$\begin{aligned}\mathbb{P}(D_3|R_1) &= \frac{\mathbb{P}(R_1|D_3)\mathbb{P}(D_3)}{\mathbb{P}(R_1|D_1)\mathbb{P}(D_1) + \mathbb{P}(R_1|D_2)\mathbb{P}(D_2) + \mathbb{P}(R_1|D_3)\mathbb{P}(D_3) + \mathbb{P}(R_1|D_4)\mathbb{P}(D_4)} \\ &= \frac{(\frac{1}{2})(\frac{1}{4})}{(0)(\frac{1}{4}) + (\frac{1}{3})(\frac{1}{4}) + (\frac{1}{2})(\frac{1}{4}) + (\frac{1}{2})(\frac{1}{4})} = \frac{3}{8}\end{aligned}$$

Switching to door 4 will give the same result: $\mathbb{P}(D_4|R_1) = \frac{3}{8}$.

The contestant will maximize the chances of winning by switching to either door 3 or door 4.